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M.A./M.Sc. (Fourth Semester)
EXAMINATION, May - June, 2022

Mathematics

Paper Third (C)

(Mathematical Biology-II)

Time : Three Hours] *[Maximum Marks:80*

[Minimum Pass Marks:16

Note: Attempt all sections as directed.

(Section - A)

(Objective/Multiple Choice Questions)

(1 mark each)

Note: Choose one correct answer out of four alternative answers.

1. A periodic age structure is known as
   (A) Leslie wave
   (B) Bernardelli population wave
   (C) Travelling wave
   (D) Turing wave

2. In the Leslie matrix population model
   (A) The survival probabilities are positive
   (B) The fecundity coefficients are non-negative
   (C) The fecundity coefficients are not all zero
   (D) All of these
3. The equation $p_a(a,t) + p_t(a,t) + \mu(a)\,p(a,t) = 0$ is known as:
   (A) The Mc-Kendrick equation
   (B) The von Foerster equation
   (C) The Lewis equation
   (D) options (A) and (B) both
4. A deterministic population model with spatial structure can be modelled by
   (A) Partial differential equations
   (B) Ordinary differential equations
   (C) Stochastic partial differential equations
   (D) Both options (A) and (B)
5. In an epidemic model, disease is eradicated (controlled) over time, if the basic reproduction number is:
   (A) Less than one
   (B) Greater than one
   (C) Equal to one
   (D) None of these

6. In a discrete population model

   (A) independent variables (time) is discrete

   (B) Dependent variables (population) is continuous

   (C) Both options (A) and (B)

   (D) None of these

7. Reaction- diffusion is used to model

   (A) Population dynamics without spatial structure

   (B) Population dynamics with spatial structure

   (C) Both options (A) and (B)

   (D) None of these

8. Tuberculosis is an infectious disease caused by:

   (A) Bacteria

   (B) Virus

   (C) Parasite

   (D) Protozoa

9. Transmissibility is an important characteristic of infectious disease. Transmissibility depends on:

   (A) Rate of contacts

   (B) The probability that a contact will transmit infection

   (C) The duration time of the infection

   (D) All of these

10. Exposed period (latent period) in an epidemic model is

    (A) After transmission of infection period from susceptible to potentially infective

    (B) After transmission of infection, period from susceptible to symptom development

    (C) Both options (A) and (B)

    (D) None of these

11. Choose the odd one

    (A) Influenza

    (B) HIV

    (C) Covid - 19

    (D) Diabetes

12. Which of the following models is used to model a disease without disease-induced immunity

    (A) SIR model

    (B) SEIR model

    (C) SIS model

    (D) SIRS model

13. What is meta-population in population dynamic

    (A) A group of homogeneous population

    (B) Group of population at a single place and same characteristics

    (C) A group of spatially separated population of the same species which interact at some level.

    (D) None of these

14. What is disease outbreak?

(A) Sudden increase in number of infections in a local region

(B) Sudden decrease in number of infections in a local region

(C) Sudden decrease in number of infections at global level

(D) All of these

15. Spatial heterogeneity is modelled by

(A) Laplace equation

(B) Heat equation

(C) Wave equation

(D) All of these

16. In India, Tuberculosis (TB) disease is currently

(A) Endemic

(B) Pandemic

(C) Both options (A) and (B)

(D) None of these

17. Routh-Hurwitz criteria is used for

(A) Local stability of linear system

(B) Global stability of linear system

(C) Local stability of nonlinear system

(D) Global stability of nonlinear system

18. Next generation matrix is used to calculate

(A) Basic reproduction number

(B) Control (Effective) reproduce on number

(C) Both options (A) and (B)

(D) None of these

19. Branching - process Disease- outbreak model is:

(A) A stochastic model

(B) A deterministic model

(C) Both options (A) and (B)

(D) None of these

20. For a very small population which type of model is more suitable:

(A) A deterministic model

(B) A stochastic model

(C) Both options (A) and (B)

(D) None of these

Section - B

(Very Short Answer Type Questions)

(2 marks each)

Note: Attempt all questions in 2 - 3 sentences.

1. What is age-structured population model? Why we need it?

2. State Perron-Frobenius theorem. Does Perron - Frobenius result hold good for all age structured population? Justify your answer not hold.
3. Define stable age distribution in discrete age- structured population.
4. Write diffusion equation and outline variable separable method.
5. What is disease free and endemic equilibria? What is the role of R_0 on their stability?
6. Describe various types of immunity and its role on infections disease dynamics.
7. Differential between deterministic *vs* Stochastic and linear *vs* non- Linear models.
8. Define M-Matrix with examples.

Section - C

(Short Answer Type Questions)

(3 marks each)

Note: Attempt all questions.

1. Describe the Leslie matrix population model and find its solution.
2. Write assumptions of the Mc-Kendrick Model (1926) for age - structured populations and derive its equation.
3. What is compartmental epidemic models? Explain with examples.
4. Define basic reproduction number (ration) in an epidemic

model and explain with suitable example.

5. Write an epidemic model with quarantine and isolation. Define all model parameters and explain the model.
6. Consider the SEIR model with infectivity in the exposed stage.

$$S^1 = -\beta S(1+\varepsilon E)$$
$$E^1 = \beta S(1+\varepsilon E) - kE$$
$$I^1 = kE - \alpha 1$$
$$R^1 = \alpha 1$$

Find the basic reproduction of the epidemic model using Next Generation Matrix method.

7. Describe Covid-19 epidemic and propose a simple model.
8. Explain herd immunity and its effects and relation with basic reproduction number.

Section - D

(Long Answer Type Questions)

(5 marks each)

Note: Attempt all questions.

1. For the Leslie matrix A, find the dominant Eigen value (if there is one), the corresponding Eigen-vectors and the stable age distribution, where

$$A = \begin{bmatrix} 0 & 1 & 1 \\ \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}$$

OR

Describe the Method of Characteristics to solve continuous linear age - structured model.

2. Consider two populations in separate patches governed by logistic equation with the same intrinsic growth rate but different carrying capacities and there is a proportional travel rate *a* from the first patch to the second patch. Model the meta population with above assumptions, find all its equilibria and their stability.

OR

Consider two populations with the same birth and death rates in two patches, with proportional travel rates in both directions, given by a system.

$$y_1' = \wedge - \mu y_1 - \sigma_1 y_1 + \sigma_2 y_2$$
$$y_2' = \wedge - \mu y_2 - \sigma_2 y_2 + \sigma_1 y_1$$

Find all biologically meaningful equilibria and analyze their stability.

3. Find a solution of the diffusion equation $U_t(x,t) = Du_{xx}(x,t) for 0 \le x \le L, 0 \le t < \infty$ subject to the boundary condition $u(0,t) = u(L,t) = 0$ and the initial con-

dition $u(x,0) = f(x), 0 \le x \le L$ with f a given smooth function.

OR

Describe residency meta-population model with two patches and no internal patch dynamics. Explain all parameters and analyze the model.

4. Describe Kermack- Mc-Kendrick Epidemic Model. Calculate basic reproduction number and derive final size relation.

OR

Describe an SIR epidemic model with birth and death. Explain all parameter. Find steady states and analyze the system.